TRINITY COLLEGE

Trinity College

Semester Two Examination, 2018

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 3 AND 4

Section Two: Calculator-assumed

SOLUTIONS

Student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	12	12	100	102	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (102 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

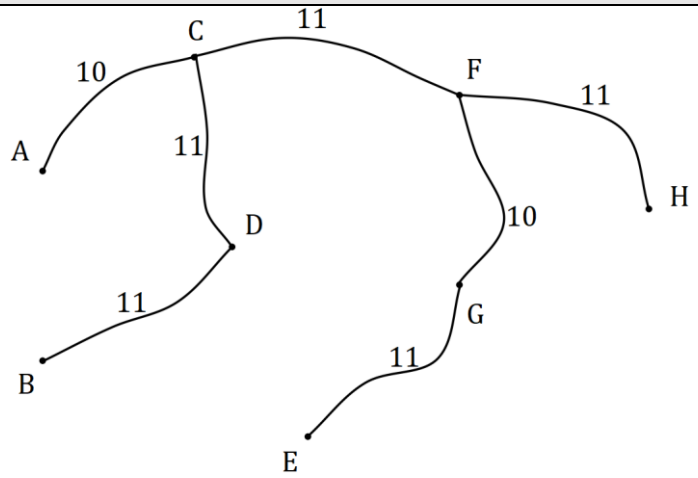
Question 8 **(6 marks)**

(a) A connected planar graph has 9 faces and 12 vertices. Determine how many edges must be removed from the graph to leave the minimum spanning tree. (3 marks)

Solution
CPG: $e = 9 + 12 - 2 = 19$ MST: $e = 1 + 12 - 2 = 11$ Hence 8 edges must be removed.
Specific behaviours
✓ indicates edges in graph ✓ indicates edges in MST ✓ states difference

(b) The vertices in the graph below represent activity centres in a theme park and the edges represent various footpaths between the centres. The weights on the edges represent the time to travel along that footpath. Identify the minimum spanning tree on the graph and state its length. (3 marks)

Solution



$L = 10 + 11 + 11 + 11 + 10 + 11 + 11 = 75$ minutes

Specific behaviours
✓ tree with 7 edges ✓ correct MST highlighted/shown ✓ correct length

See next page

Question 9 **(6 marks)**

The deseasonalised number of working holiday makers in Australia over the four-year period from March 2014 to December 2017 was modelled by $n = 160.74 - 1.382t$, where n is the number of people in thousands and t is the quarter, with $t = 1$ corresponding to March 2014.

The Seasonal Index table is shown below.

Quarter	March	June	September	December
Seasonal Index	1.07	x	0.96	1.02

(a) Determine the value of x in the table above. (1 mark)

Solution
$x = 4 - 1.07 - 0.96 - 1.02 = 0.95$
Specific behaviours
✓ correct value

(b) State, with reasons, whether the number of working holiday makers in Australia was highest during September 2017 or during September 2014. (2 marks)

Solution
Sept 2014, as the gradient of the linear model is -1.382, indicating a decrease of 1382 people per quarter.
Specific behaviours
✓ chooses correct year ✓ indicates gradient implies decrease over time

(c) Use your knowledge of time series to estimate the actual change in the number of working holiday makers in Australia from December 2017 ($t = 16$) to March 2018 ($t = 17$). (3 marks)

Solution
$[160.74 - 1.382(16)] \times 1.02 = 138.6 \times 1.02 = 141.40$ $[160.74 - 1.382(17)] \times 1.07 = 137.2 \times 1.07 = 146.85$ $146.9 - 141.4 = 5.45 \Rightarrow \text{Increase of } 5\,450 \text{ people}$
Specific behaviours
✓ correct prediction for $t = 16$ in 000's ✓ correct prediction for $t = 17$ in 000's ✓ correct change, adjusting for 000's

See next page

Question 10 **(8 marks)**

The data in the table below was collected by a student who was investigating whether an association exists between a person's hair and eye colour. The observations were taken from a survey of 73 people.

		Eye colour	
		Blue-Green	Brown
Hair colour	Black	0	13
	Blond	18	14
	Brown	5	23

(a) What percentage of the people surveyed had blond hair? (1 mark)

Solution
$32 \div 73 = 44\%$
Specific behaviours
✓ correct percentage

(b) What percentage of the people with blue-green eyes had brown hair? (1 mark)

Solution
$5 \div 23 = 22\%$
Specific behaviours
✓ correct percentage

(c) Complete the table of **column** percentages below. (2 marks)

		Eye colour	
	%	Blue-Green	Brown
Hair colour	Black	0	26
	Blond	78	28
	Brown	22	46

Solution
See table
Specific behaviours
✓ one column correct
✓ both columns correct

(d) Does the data suggest the presence of an association between the categorical variables? Justify your answer using figures from the percentage table. (2 marks)

Solution
Yes, as the two percentages in each row are quite different. For example, 46% of brown eyed people have brown hair but only 22% of those with blue-green eyes have brown hair.
Specific behaviours
✓ yes to association
✓ uses appropriate percentages to justify

(e) One of the conclusions made by the student was that having black hair caused a person to have brown eyes. Comment on this conclusion. (2 marks)

Solution
This conclusion is not valid. An observed association does not mean there is a causal relationship between the variables.
Specific behaviours
✓ indicates conclusion not valid
✓ comments on causality

See next page

Question 11 **(10 marks)**

At the start of March an annuity was set up with a sum of $\$495\,000$. At the end of each month, interest on the balance at the start of the month was added and then $\$4\,250$ was withdrawn. The table below illustrates this process.

Month, n	Balance at start of month, T_n	Interest for month	Withdrawal	Balance at end of month, T_{n+1}
1	\$495 000.00	\$3 217.50	\$4 250.00	\$493 967.50
2	\$493 967.50	\$3 210.79	\$4 250.00	\$492 928.29
3	\$492 928.29	\$3 204.03	\$4 250.00	\$491 882.32
4	\$491 882.32	\$3 197.24	\$4 250.00	D
5		E		

(a) Show how to use values from the table to deduce that the annual interest rate is 7.8%. (2 marks)

Solution
Monthly rate: $3\,217.50 \div 495\,000 = 0.0065$ Annual rate as %: $0.0065 \times 12 \times 100 = 7.8\%$
Specific behaviours
✓ shows monthly rate calculation ✓ shows conversion to annual rate calculation

(b) The linear recurrence relation for the balance of the annuity at the start of month n has the form $T_{n+1} = aT_n - b,\ T_1 = c$. State the values of a, b and c. (3 marks)

Solution
$a = 1.0065, \quad b = 4\,250, \quad c = 495\,000$
Specific behaviours
✓ value of a ✓ value of b ✓ value of c

(c) Determine the values of D and E in the table. (2 marks)

Solution
$D = \$490\,829.56, \quad Q = \$3\,190.39$
Specific behaviours
✓ value of D ✓ value of E

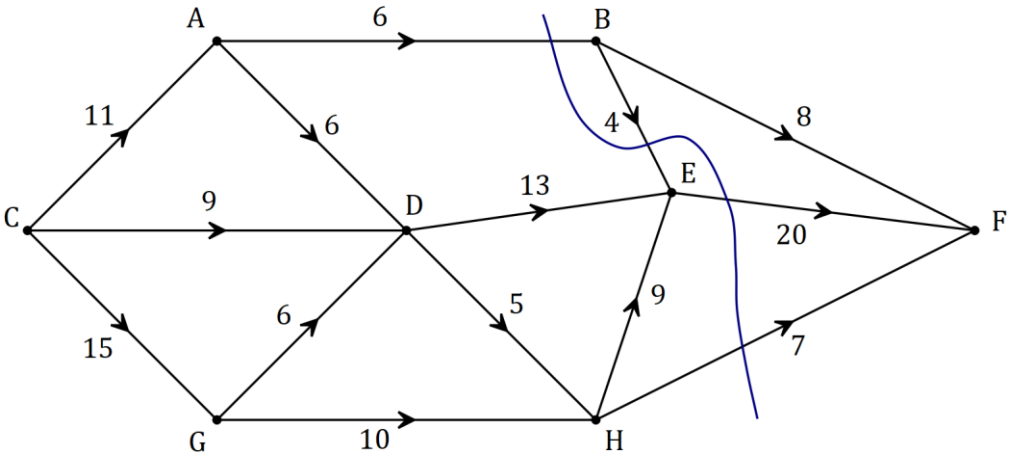
(d) Determine the balance of the annuity at the end of month 150 and calculate the total interest that the annuity has earned over the first 150 months. (3 marks)

Solution
$T_{151} = \$234\,045.04$ $I = 234\,045.04 + 150 \times 4\,250 - 495\,000 = \$376\,545.04$
Specific behaviours
✓ correct balance ✓ calculates total withdrawals ✓ correct total interest

See next page

Question 12 **(6 marks)**

Spectators leave a sports ground C and walk to a train station F along footpaths in the directions shown on the network below. The weights on the edges represent the maximum number of people who can safely travel along each footpath, in hundreds of people per minute.



(a) By listing the different paths and their corresponding flow rates, determine the maximum number of people that can walk through the network from C to F every minute. (4 marks)

Solution
CABF = 6 CADEF = 5 CDEF = 8 CDHEF = 1 CGDHEF = 4 CGHEF = 2 CGHF = 7 Total flow = 33, hence maximum of 3 300 people per minute.
Specific behaviours
✓ correctly shows at least two paths with flow contributions ✓ correctly shows all paths with flow contributions ✓ correct total flow ✓ correctly states the maximum flow in people per minute

(b) Verify your answer from part (a) by showing the minimum cut on the network above and showing in the space below how to determine the capacity of the cut. (2 marks)

Solution
See graph. NB Flow $B \rightarrow E$ is reversed. Capacity of cut: $6 + \cancel{(-4)} + 20 + 7 = 33$
Specific behaviours
✓ cut clearly shown on graph ✓ correct values used to obtain capacity

See next page

Question 13 **(12 marks)**

To investigate the hypothesis that the mass of sugar, w grams, that will dissolve in 300 ml of water changes with the temperature, $t°C$, of the water, a student collected the results shown in the table below.

t	20	25	30	35	40	45	50	55	60
w	151	210	241	248	292	309	347	376	433

(a) Identify the response variable in this investigation. (1 mark)

Solution
Mass of sugar
Specific behaviours
✓ correct variable

(b) Calculate the correlation coefficient between temperature and mass of dissolved sugar and hence describe the direction and strength of the association between the variables. (3 marks)

Solution
$r = 0.990$ Association is strong and positive.
Specific behaviours
✓ value to at least 2dp ✓ direction ✓ strength

(c) Determine the equation of the least-squares line for predicting w from t. (2 marks)

Solution
$w = 6.33t + 36.5$
Specific behaviours
✓ gradient ✓ y-intercept

(d) Interpret, in context, the value of

(i) the gradient of the least-squares line in (c). (1 mark)

Solution
For every $1°C$ increase in water temperature, an extra 6.33 grams of sugar will dissolve.
Specific behaviours
✓ correct interpretation

(ii) the y-intercept of the least-squares line in (c). (1 mark)

Solution
36.5 is the mass of sugar that will dissolve when the temperature of the water is $0°C$.
Specific behaviours
✓ correct interpretation

See next page

(e) Predict the mass of sugar that will dissolve in 300 ml of water at a temperature of

(i) $78°C$. (1 mark)

Solution
$w = 530$ g
Specific behaviours
✓ correct value

(ii) $23°C$. (1 mark)

Solution
$w = 182$ g
Specific behaviours
✓ correct value

(f) Comment on the reliability of the prediction in (e)(ii). Justify your answer. (2 marks)

Solution
The second prediction for $23°C$, is very reliable due to strong correlation and interpolation.
Specific behaviours
✓ rates as reliable ✓ with reasoning

Question 14 **(12 marks)**

Aya deposits a fixed sum in her bank account each week so that its balance after n deposits is given by A_n, where $A_{n+1} = A_n + \$3.50$, $A_1 = \$59$.

(a) Determine

(i) the balance of Aya's account after 20 deposits. (1 mark)

Solution
$A_{20} = \$125.50$
Specific behaviours
✓ correct amount

(ii) the least number of deposits Aya must make so that the balance in her account exceeds $\$200$. (1 mark)

Solution
$n = 42$
Specific behaviours
✓ correct number

Starting at the same time as Aya, Bart withdraws a fixed sum from his bank account every week so that its balance after n withdrawals is given by B_n, where $B_{n+1} = B_n - \$1.80$, $B_1 = \$324$.

(b) Determine

(i) the n^{th} term rule for the balance of Bart's account after n withdrawals. (2 marks)

Solution
$B_n = 324 - 1.8(n-1)$
Specific behaviours
✓ correct form; value of first term
✓ correctly shows $(n-1)$ multiplied by -1.8

(ii) the maximum number of withdrawals Bart can make until he has no money left. (1 mark)

Solution
$n = 181$
Specific behaviours
✓ correct number

(c) Determine the value of n so that $A_n = B_n$ and state the value of A_n at this time. (2 marks)

Solution
$n = 51$ $A_n = \$234$
Specific behaviours
✓ correct value of n
✓ correct value of A_n

See next page

(d) Let C_n be the combined balance, in dollars, that Aya and Bart have in their accounts after they have made n deposits and n withdrawals respectively.

(i) Show that $C_5 = \$389.80$. (1 mark)

Solution
$A_5 = 73, \quad B_5 = 316.80$ $C_5 = 73 + 316.80 = 389.80$
Specific behaviours
✓ shows values of A_5 and B_5

(ii) The n^{th} term rule for C_n is $C_n = an + b$. Determine the values of a and b. (2 marks)

Solution
$C_1 = 59 + 324 = 383$ $C_2 = 383 + 3.50 - 1.80 = 383 + 1.70$ $C_n = 383 + 1.7(n-1)$ $= 383 + 1.7n - 1.7$ $= 381.3 + 1.7n$ $a = 1.7, \quad b = 381.3$
Specific behaviours
✓ indicates first term and common difference of sequence ✓ correct values

(iii) Determine the smallest value of n for C_n to exceed $\$525$. (2 marks)

Solution
$525 = 381.3 + 1.7n$ $n = 84.53$ $\therefore\ n = 85$

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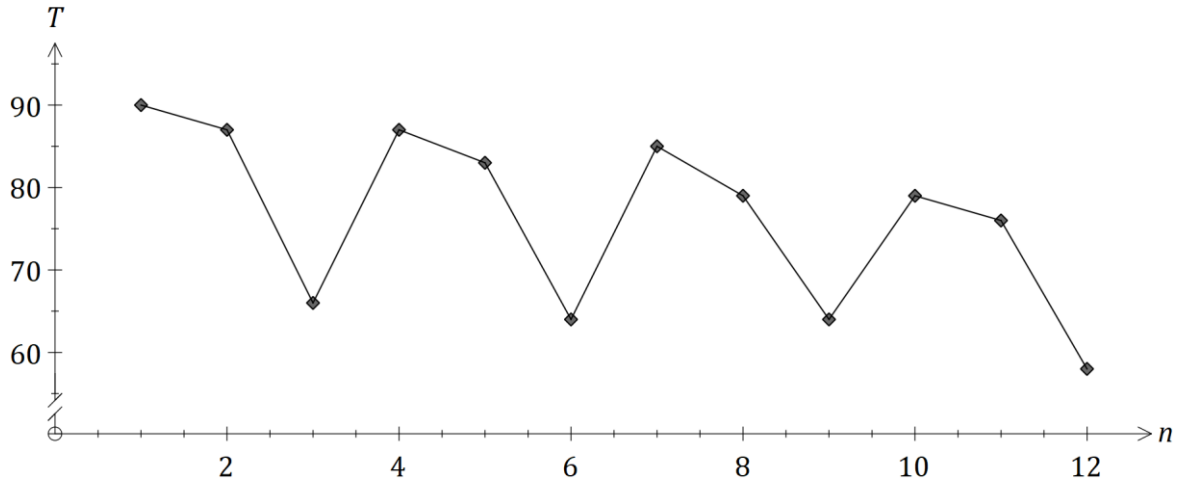
Question 15 **(10 marks)**

The time, T minutes, a new driver spent delivering goods on the same city route is shown in the table and graph below.

Week	Day	Day number (n)	Time (T)	3-day mean	Percentage of 3-day mean	Deseasonalised time (t)
1	Fri	1	90		111.1	80.8
1	Sat	2	87	81	107.4	81.9
1	Sun	3	66		81.5	80.1
2	Fri	4	87		111.5	78.1
2	Sat	5	83	A	106.4	78.2
2	Sun	6	64		82.1	77.7
3	Fri	7	85		B	76.3
3	Sat	8	79	76	103.9	74.4
3	Sun	9	C		84.2	77.7
4	Fri	10	79		111.3	D
4	Sat	11	76	71	107	71.6
4	Sun	12	58		81.7	70.4



(a) Calculate the values of the entries A, B and C in the table. (3 marks)

Solution
$A = \frac{87 + 83 + 64}{3} = 78$ $B = \frac{85}{76} \times 100 = 111.8$ $C = 84.2 \times 76 \div 100 = 64$
Specific behaviours
✓ value of A ✓ value of B ✓ value of C

See next page

(b) Complete the Seasonal Index table below. (2 marks)

Day	Fri	Sat	Sun
Seasonal Index	1.114	1.062	0.824

Solution
$Sat = \dfrac{107.4 + 106.4 + 103.9 + 107}{4 \times 100} = 1.062$ $Sun = 3 - 1.114 - 1.062 = 0.824$
Specific behaviours
✓ one correct index ✓ both correct

(c) Calculate D, the deseasonalised value of T for Friday of Week 4. (2 marks)

Solution
$t = 79 \div 1.114 = 70.9$
Specific behaviours
✓ uses correct value of T and index ✓ correct value of t

The equation of the least-squares line for t against n is $\hat{t} = 83.0 - 1.001n$.

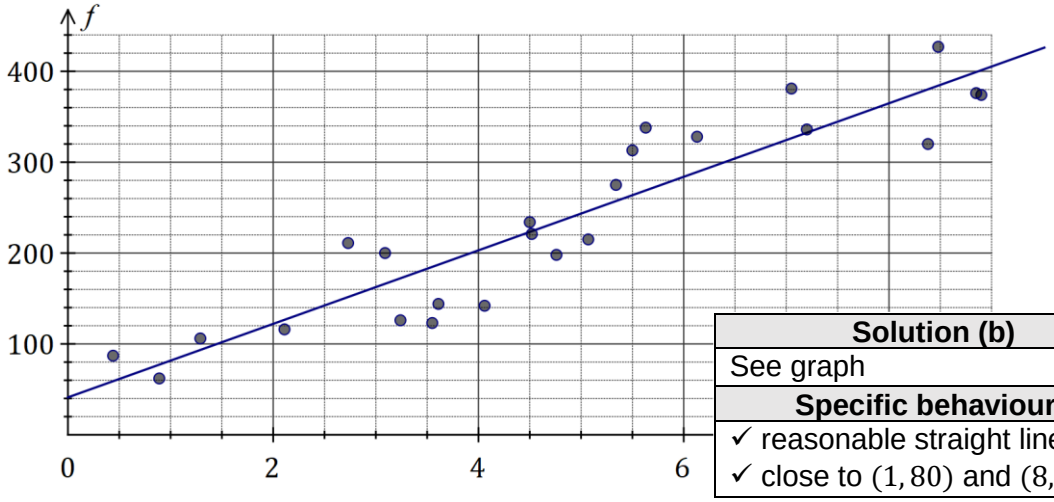
(d) Forecast the time T that the driver will take on Friday of Week 5 if the existing trend and seasonality continue. (3 marks)

Solution
$n = 13$ $\hat{t} = 83.0 - 1.001(13) = 70.0$ $T = 70.0 \times 1.114 = 78 \text{ minutes}$
Specific behaviours
✓ correct value of n ✓ correct value of $\hat{t}$ ✓ correct forecast for T

See next page

Question 16 **(8 marks)**

A company recorded the distance travelled, k in hundreds of km, and the amount of fuel used, f litres, for the trucks in its fleet each day. The scatterplot below shows the data for one day, for which $r = 0.927$ and the equation of the least-squares line is $f = 40.5k + 41.2$.



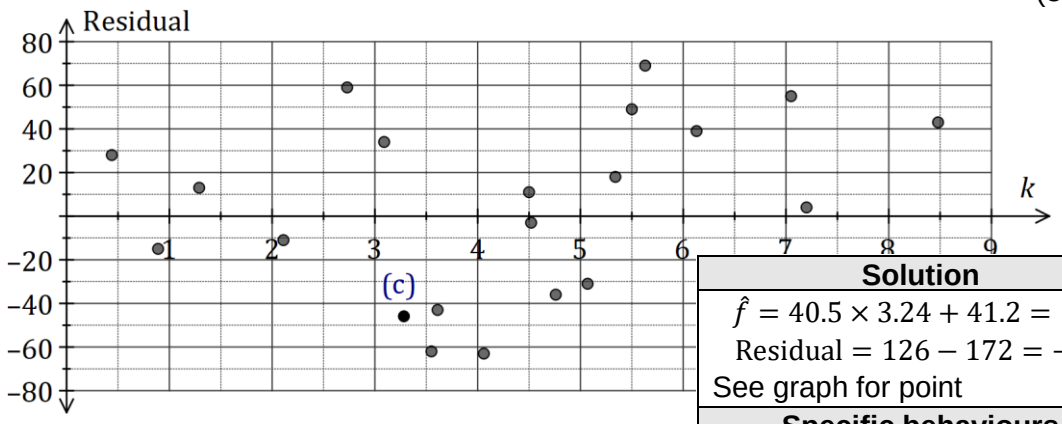
Solution (b)
See graph
Specific behaviours
✓ reasonable straight line
✓ close to $(1, 80)$ and $(8, 360)$

(a) What percentage of the variation in the amount of fuel used can be explained by the variation in distance travelled? (1 mark)

Solution
$r^2 = 0.927^2 = 0.86 \Rightarrow 86\%$
Specific behaviours
✓ correct percentage

(b) Draw the least-squares line on the scatterplot above. (2 marks)

(c) Determine the residual for the data point $(3.24, 126)$ and add it to the residual plot below. (3 marks)



Solution
$\hat{f} = 40.5 \times 3.24 + 41.2 = 172$
$\text{Residual} = 126 - 172 = -46$
See graph for point
Specific behaviours
✓ calculates $\hat{f}$
✓ calculates residual
✓ plots accurately

(d) Use the residual plot to comment on the appropriateness of fitting a linear model to the data. (2 marks)

Solution
Linear model IS appropriate, as no pattern evident in the residuals.
Specific behaviours
✓ indicates appropriate
✓ supplies suitable reason

See next page

Question 17 **(8 marks)**

(a) A company advertised compound interest of 14.3% pa on investments of $\$350\,000$ for a period of 4 years. Calculate the total interest on the investment over this time. (2 marks)

Solution
$I = 350\,000(1.143)^4 - 350\,000$ $= \$247\,383.15$
Specific behaviours
✓ indicates calculation for future value ✓ correct interest

(b) Savings account A offers interest of 3.83% pa compounded quarterly and savings account B offers interest of 3.82% pa compounded monthly. Calculate the effective interest rate for both accounts and hence decide which savings account offers the better return. (3 marks)

Solution
Account A: $(1 + 0.0383)^4 - 1 = 0.03\,885 = 3.885\%$ Account B: $(1 + 0.0382)^{12} - 1 = 0.03\,888 = 3.888\%$ Account B offers the better return.
Specific behaviours
✓ EIR for A ✓ EIR for B ✓ indicates account giving better return

(c) A philanthropist is considering funding 12 scholarships to a local college. Each student in receipt of a scholarship would receive a payment of $\$5\,000$ on the yearly anniversary of the creation of the fund. Determine, to the nearest $\$100$, the initial sum of money that should be deposited in an account paying interest at a rate of 2.8% compounded monthly to create a perpetuity to fund all 12 scholarships. (3 marks)

Solution
$\text{EIR} = (1 + 0.028 \div 12)^{12} - 1 = 0.028362$ $I = 12 \times 5\,000 = 60\,000$ $P \times 0.028362 = 60\,000$ $P \approx \$2\,115\,500$
Specific behaviours
✓ calculates effective interest rate ✓ calculates interest required ✓ solves for principal

See next page

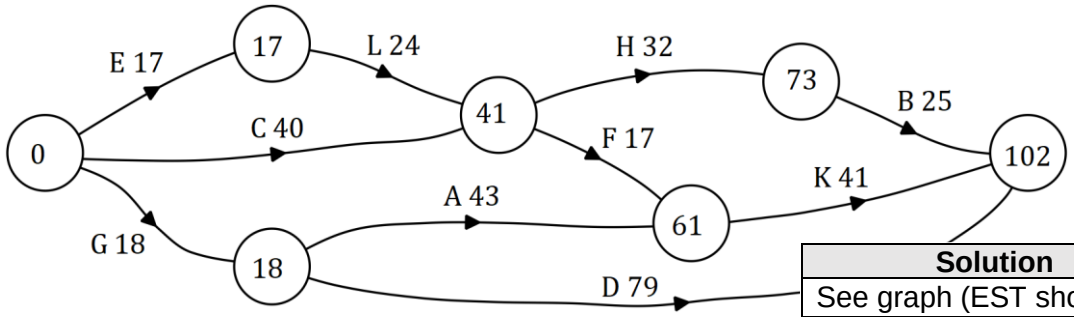
Question 18 **(7 marks)**

A project requires the following 10 activities to be completed.

Activity	Immediate predecessor(s)	Time (days)
A	G	43
B	H	25
C	None	40
D	G	79
E	None	17

Activity	Immediate predecessor(s)	Time (days)
F	C, L	17
G	None	18
H	C, L	32
K	A, F	41
L	E	24

(a) The network below represents the interdependencies of the above activities. Clearly label each edge with its activity and time. (3 marks)



Solution
See graph (EST shown)
Specific behaviours
✓ identifies edges C, E, G
✓ at least 7 correct labels
✓ all labels correct

(b) Identify the critical path and hence state the minimum time for the project to be completed. (2 marks)

Solution
CP: $\mathrm{G} - A - K$ $\mathrm{MCT} = 18 + 43 + 41 = 102$ days
Specific behaviours
✓ identifies critical path
✓ states MCT

(c) Determine the latest start time and float time for activity H. (2 marks)

Solution
$LST = 102 - 25 - 32 = \mathrm{Day}\ 45$ $\mathrm{Float} = 45 - 41 = 4$ days
Specific behaviours
✓ latest start time
✓ float time

See next page

Question 19 **(9 marks)**

A business loan of $\$120\,000$ was taken out at the start of the first quarter of 2018. Interest, at a rate of 13.8% per annum, was calculated on the balance of the loan on the last day of each month and added to the loan. Repayments of $\$12\,410.64$ were made on the last day of each quarter, just after interest for the month was added.

(a) Calculate the interest added during the first quarter of 2018. (3 marks)

Solution
Monthly rate: $r = 0.138 \div 12 = 0.0115$ After 3 months: $120\,000 \times 1.0115^3 = 124\,187.79$ Total: $I = 124\,187.79 - 120\,000 = \$4\,187.79$
Specific behaviours
✓ indicates monthly interest rate ✓ indicates suitable method ✓ correct interest

(b) Determine

(i) the balance of the loan at the start of the second quarter of 2018. (1 mark)

Solution
$\text{Balance} = 120\,000 + 4\,187.79 - 12\,410.64 = \$111\,777.15$
Specific behaviours
✓ correct balance

(ii) the number of repayments to fully repay the loan. (1 mark)

Solution
Using financial calculator, $N = 12$
Specific behaviours
✓ correct number

(iii) the total interest charged over the life of the loan, to the nearest dollar. (2 marks)

Solution
$I = 12 \times 12\,410.64 - 120\,000 = \$28\,927.68$ $I \sim \$28\,928$
Specific behaviours
✓ indicates total repaid ✓ correct interest (no penalty if not to nearest dollar)

(c) The business decided to halve the loan repayments, assuming that it would take twice as long to repay the loan. Write a brief note to the business commenting on the validity of this assumption. (2 marks)

Solution
Assumption not true because balances owing at the end of each quarter will be greater and so there will be additional interest, taking 33 quarters to repay the loan.
Specific behaviours
✓ explanation referring to extra time because of extra interest ✓ states actual time to repay loan

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____